Year 6 algebra

Prior Knowledge
This topic is first introduced in Year 6 but sequencing and patterns have featured in every year group.

algebra		Working Towards	Within	Expected	Above
$2y + x$	Use simple formulae				
	Generate and describe linear number sequences				
	Express missing number problems algebraically				
	Find pairs of numbers that satisfy an equation with two unknowns				
	Enumerate possibilities of combinations of 2 variables				

Highlights: _____

_____

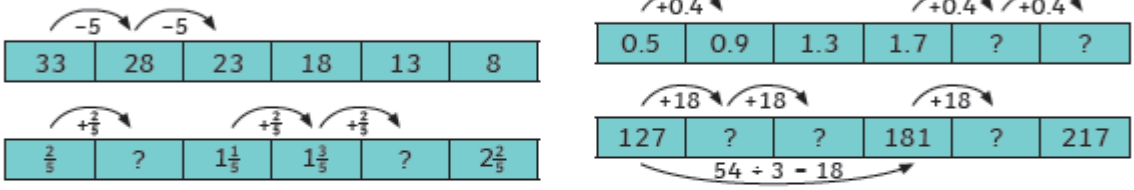
Glossary

vocabulary	word class	definition
formulae	plural noun	a mathematical relationship or rule expressed in symbols
linear	adjective	arranged in or extending along a straight or nearly straight line
algebra	noun	the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations
equation	noun	a statement that the values of two mathematical expressions are equal (indicated by the sign =)
enumerate	verb	establish the number of
variable	adjective	able to assume different numerical values

Linear Number Sequences

A linear number sequence is a sequence where each value increases or decreases by the same amount each time. Each number in a linear number sequence is called a **term**. The constant change between each number is called the term to term rule. To identify the **term to term rule**, find the difference between two adjacent terms.

When you know the term to term rule, you can use it to find the next number in the sequence. It can also be used to find a missing number within a sequence.



Forming Expressions

An expression is a group of numbers, letters and operation symbols.

Add 14 to a	$a + 14$
Subtract 20 from b	$b - 20$
Multiply c by 4	$4c$
12 more than d	$d + 12$
Multiply e by 3 and subtract 5	$3e - 5$
Add 12 to f and then multiply by 2	$2(f + 12)$

Forming Equations

$a + 14 = 20$
$b - 20 = 15$
$4c = 28$
$d + 12 = 30$
$3e - 5 = 10$
$2(f + 12) = 44$

An equation is a number statement with an equal sign (=). Expressions on either side of the equal sign are of equal value.

Equations with Pairs of Unknowns

In an equation with two unknown numbers, there may be **several** possible values for the unknowns that will balance the equation.

$ab = 18$

a	b
1	18
2	9
3	6
6	3
9	2
18	1

$2a + b = 10$

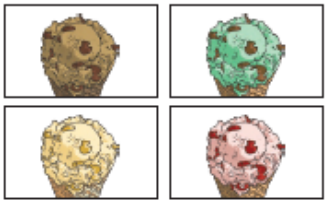
a	b
2	6
3	4
4	2
5	0

Enumerating Possibilities

Enumerating means making a complete list of answers to a problem.

- Use a system for finding the possibilities.
- Organise your findings in an ordered list or table.
- Have a way of deciding when all possibilities have been found.

There are four ice cream flavours.



Two scoops of two different flavours give six possible combinations.

- chocolate and strawberry
- chocolate and vanilla
- chocolate and mint
- strawberry and vanilla
- strawberry and mint
- vanilla and mint

Future Learning

Key Stage 3

- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- use and interpret algebraic notation, including:

ab in place of $a \times b$
$3y$ in place of $y + y + y$ and $3 \times y$
$a2$ in place of $a \times a$, $a3$ in place of $a \times a \times a$; $a2b$ in place of $a \times a \times b$
ba in place of $a \div b$
coefficients written as fractions rather than as decimals
brackets
substitute numerical values into formulae and expressions, including scientific formulae
understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
simplify and manipulate algebraic expressions to maintain equivalence by:
collecting like terms
multiplying a single term over a bracket
taking out common factors
expanding products of two or more binomials

- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the nth term
- recognise geometric sequences and appreciate other sequences that arise

Key Stage 4

- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions, {**and expressions involving surds and algebraic fractions**}
- extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, {**exponential and trigonometric**} functions
- use mathematical language and properties precisely.
- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
- extend their ability to identify variables and express relations between variables algebraically and graphically
- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments {**and proofs**}
- reason deductively in geometry, number and algebra, including using geometrical constructions
- In addition to consolidating subject content from key stage 3, pupils should be taught to:
  simplify and manipulate algebraic expressions (including those involving surds {**and algebraic fractions**}) by:
  factorising quadratic expressions of the form $x2 + bx + c$
  $\boldsymbol{ax2 + bx + c}$
  including the difference of two squares; {**factorising quadratic expressions of the form** }
  simplifying expressions involving sums, products and powers, including the laws of indices
  know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments {**and proofs**}
  where appropriate, interpret simple expressions as functions with inputs and outputs; {**interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'**}
  use the form
  $=+ymxc$ to identify parallel {**and perpendicular**} lines; find the equation of the line through two given points, or through one point with a given gradient
  identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically {**and turning points by completing the square**}
  recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = 1$
  x
  $\boldsymbol{y = \cos x}$
  with $x \neq 0$, {**the exponential function**
  $\boldsymbol{y = kx}$
  $\boldsymbol{y = \sin x}$
  for positive values of *k*, and the trigonometric functions (with arguments in degrees) , and $y = \tan x$ for angles of any size}
  {**sketch translations and reflections of the graph of a given function**}
- plot and interpret graphs (including reciprocal graphs {**and exponential graphs**}) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- {**calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts**}
- {**recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**}
- solve quadratic equations {**including those that require rearrangement**} algebraically by factorising, {**by completing the square and by using the quadratic formula**}; find approximate solutions using a graph
- solve two simultaneous equations in two variables (linear/linear {**or linear/quadratic**}) algebraically; find approximate solutions using a graph
- {**find approximate solutions to equations numerically using iteration**}
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- solve linear inequalities in one {**or two**} variable{**s**}**,** {**and quadratic inequalities in one variable**}; represent the solution set on a number line, {**using set notation and on a graph**}
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number {**or a surd**}) {**and other sequences**}
- deduce expressions to calculate the n^{th} term of linear {**and quadratic**} sequences